

Neutrino Mixing Angles from Texture Zeros of the Lepton Mass Matrices

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Abstract

Taking into account the latest neutrino oscillation data, we study texture zeros of the lepton mass matrices. Assuming the Dirac neutrino mass matrix M_D , the charged-lepton mass matrix M_l and the mass matrix of heavy right-handed Majorana neutrinos M_R to have three texture zeros, we show that the observed neutrino mixing angles can naturally be obtained. The phenomenological implications for the neutrino mass spectrum, the CP-violating phases, the tritium beta decay and the neutrinoless double-beta decay are explored.

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1 Introduction

The recent Daya Bay [1] and RENO [2] reactor neutrino experiments reveal that the smallest neutrino mixing angle is relatively large (i.e., $\theta_{13} \approx 9^\circ$). The solar and atmospheric neutrino oscillation experiments indicate that the other two mixing angles are large (i.e., $\theta_{12} \approx 34^\circ$ and $\theta_{23} \approx 40^\circ$). This has been confirmed by the long-baseline accelerator and reactor neutrino oscillation experiments. Two independent neutrino mass-squared differences have been determined: $\delta m^2 \equiv m_2^2 - m_1^2 \approx 7.5 \times 10^{-5} \text{ eV}^2$ and $\Delta m^2 \equiv m_3^2 - (m_1^2 + m_2^2)/2 \approx \pm 2.4 \times 10^{-3} \text{ eV}^2$, where m_i (for $i = 1, 2, 3$) are the neutrino masses. The next important step in neutrino physics will be the determination of the sign of Δm^2 and the measurement of the leptonic CP-violating phase.

It has been shown that the texture zeros in the quark mass matrices [3]

$$M_q = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad (1)$$

give successful relations between the quark mixing angles and the quark mass ratios. The texture zeros were used for the lepton mass matrices to explain the observed bi-large neutrino mixing pattern, given a weakly-hierarchical neutrino mass spectrum (e.g., $m_1 : m_2 : m_3 \approx 1 : 3 : 10$) [4, 5].

We shall reexamine the texture zeros and confront them with the recent neutrino oscillation data. It turns out the original model, where both the charged-lepton and effective neutrino mass matrices assume the same texture zeros, has been already ruled out, since the observed relatively large mixing angle θ_{13} requires a large ratio of two neutrino mass-squared differences, which is not allowed by the current data. We consider the canonical seesaw model and use the texture zeros for the Dirac neutrino mass matrix M_D , the charged-lepton mass matrix M_l , and the mass matrix of the heavy right-handed Majorana neutrinos M_R . Then the effective neutrino mass matrix is given by the well-known seesaw formula $M_\nu = M_D M_R^{-1} M_D^T$, and both a nonmaximal θ_{23} and an unsuppressed θ_{13} can be naturally accommodated.

This paper is organized as follows. In Sec. 2 we apply the texture zeros to the effective neutrino mass matrix as well as the charged lepton mass matrix. We explain why this scenario is now disfavored by current neutrino oscillation data. Sec. 3 is devoted to a canonical seesaw model with texture zeros of the lepton mass matrices. Two models of the effective neutrino mass matrix have been discussed in detail. The phenomenological implications are explored, including the neutrino mixing angles, the neutrino mass spectrum, the leptonic CP violation, the tritium beta decay and the neutrinoless double-beta decays. We summarize the conclusions in Sec. 4.

2 Effective Neutrino Mass Matrix

The lepton mass spectra and mixing parameters are determined by the lepton mass terms

$$-\mathcal{L}_m = \overline{(e_L \quad \mu_L \quad \tau_L)} M_l \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} + \frac{1}{2} \overline{(\nu_{eL} \quad \nu_{\mu L} \quad \nu_{\tau L})} M_\nu \begin{pmatrix} \nu_{eL}^C \\ \nu_{\mu L}^C \\ \nu_{\tau L}^C \end{pmatrix} + \text{h.c.}, \quad (2)$$

with $\nu_{\alpha L}^C \equiv C \overline{\nu_{\alpha L}}^T$ (for $\alpha = e, \mu, \tau$), where M_l and M_ν stand for the charged-lepton mass matrix and the effective neutrino mass matrix. Since M_ν is a symmetric complex matrix, we take M_l to be

symmetric as well. Furthermore we assume that both M_ν and M_l are of the following form:

$$M_f = \begin{pmatrix} 0 & \mathcal{C}_f & 0 \\ \mathcal{C}_f & 0 & \mathcal{B}_f \\ 0 & \mathcal{B}_f & \mathcal{A}_f \end{pmatrix}, \quad (3)$$

where \mathcal{C}_f and \mathcal{B}_f are in general complex, while \mathcal{A}_f can be made real by removing the overall phase of M_f . Since both M_l and M_ν are symmetric, they can be diagonalized through a unitary transformation $U_f^\dagger M_f U_f^* = \text{Diag}\{\lambda_1^f, \lambda_2^f, \lambda_3^f\}$, where λ_i^f (for $i = 1, 2, 3$) denote the lepton mass eigenvalues. After the diagonalization of M_l and M_ν the lepton mixing matrix is given by $V = U_l^\dagger U_\nu$. The diagonalization can be done as follows. The mass matrix M_f in Eq. (3) can be decomposed into $M_f \equiv P_f \overline{M}_f P_f^T$ with

$$\overline{M}_f = \begin{pmatrix} 0 & C_f & 0 \\ C_f & 0 & B_f \\ 0 & B_f & A_f \end{pmatrix} \quad (4)$$

and $P_f = \text{Diag}\{e^{i(\varphi_f - \phi_f)}, e^{i\phi_f}, 1\}$, where $A_f = \mathcal{A}_f$, $B_f = |\mathcal{B}_f|$, $C_f = |\mathcal{C}_f|$, $\phi_f = \arg[\mathcal{B}_f]$ and $\varphi_f = \arg[\mathcal{C}_f]$. Then the real and symmetric matrix \overline{M}_f is diagonalized:

$$(O_f Q)^T \overline{M}_f (O_f Q) = \begin{pmatrix} \lambda_1^f & 0 & 0 \\ 0 & \lambda_2^f & 0 \\ 0 & 0 & \lambda_3^f \end{pmatrix}, \quad (5)$$

where $Q = \text{Diag}\{1, i, 1\}$ has been introduced to cancel the minus sign of $\text{Det}[\overline{M}_f] = -A_f C_f^2$, and O_f is a real orthogonal matrix. Therefore the nonzero matrix elements of \overline{M}_f can be expressed in terms of the eigenvalues λ_i^f (for $i = 1, 2, 3$):

$$\begin{aligned} A_f &= \lambda_1^f - \lambda_2^f + \lambda_3^f, \\ B_f &= \left[\frac{(\lambda_1^f - \lambda_2^f)(\lambda_2^f - \lambda_3^f)(\lambda_1^f + \lambda_3^f)}{\lambda_1^f - \lambda_2^f + \lambda_3^f} \right]^{1/2}, \\ C_f &= \left(\frac{\lambda_1^f \lambda_2^f \lambda_3^f}{\lambda_1^f - \lambda_2^f + \lambda_3^f} \right)^{1/2}. \end{aligned} \quad (6)$$

The matrix elements of the orthogonal matrix O_f are:

$$\begin{aligned} O_{11}^f &= + \left[\frac{x_f - z_f}{(1 + x_f)(1 - z_f)(x_f - z_f + x_f z_f)} \right]^{1/2}, \\ O_{12}^f &= - \left[\frac{x_f^3 (1 + z_f)}{(1 + x_f)(x_f + z_f)(x_f - z_f + x_f z_f)} \right]^{1/2}, \\ O_{13}^f &= + \left[\frac{z_f^3 (1 - x_f)}{(1 - z_f)(x_f + z_f)(x_f - z_f + x_f z_f)} \right]^{1/2}, \\ O_{21}^f &= + \left[\frac{x_f - z_f}{(1 + x_f)(1 - z_f)} \right]^{1/2}, \end{aligned}$$

$$\begin{aligned}
O_{22}^f &= + \left[\frac{x_f (1 + z_f)}{(1 + x_f)(x_f + z_f)} \right]^{1/2}, \\
O_{23}^f &= + \left[\frac{z_f (1 - x_f)}{(1 - z_f)(x_f + z_f)} \right]^{1/2}, \\
O_{31}^f &= - \left[\frac{x_f z_f (1 - x_f)(1 + z_f)}{(1 + x_f)(1 - z_f)(x_f - z_f + x_f z_f)} \right]^{1/2}, \\
O_{32}^f &= - \left[\frac{z_f (1 - x_f)(x_f - z_f)}{(1 + x_f)(x_f + z_f)(x_f - z_f + x_f z_f)} \right]^{1/2}, \\
O_{33}^f &= + \left[\frac{x_f (1 + z_f)(x_f - z_f)}{(1 - z_f)(x_f + z_f)(x_f - z_f + x_f z_f)} \right]^{1/2},
\end{aligned} \tag{7}$$

where we have defined $x_f \equiv \lambda_1^f/\lambda_2^f$ and $z_f \equiv \lambda_1^f/\lambda_3^f$. For the charged leptons we have: $x_l = m_e/m_\mu \approx 4.84 \times 10^{-3}$ and $z_l = m_e/m_\tau \approx 2.87 \times 10^{-4}$. For the neutrinos we have: $x_\nu = m_1/m_2$ and $z_\nu = m_1/m_3$. From Eq. (7) one finds that only the normal neutrino mass hierarchy with $z_\nu < x_\nu < 1$ is allowed. Therefore M_l is diagonalized through $U_l^\dagger M_l U_l^* = \text{Diag}\{m_e, m_\mu, m_\tau\}$ with $U_l = P_l O_l Q^*$, while M_ν through $U_\nu^\dagger M_\nu U_\nu^* = \text{Diag}\{m_1, m_2, m_3\}$ with $U_\nu = P_\nu O_\nu Q^*$. Except for the phases $\varphi_{l,\nu}$ and $\phi_{l,\nu}$ the unitary matrices U_l and U_ν are determined by the mass ratios of charged leptons and neutrinos. For the lepton mixing matrix $V = U_l^\dagger U_\nu$ the absolute values of the matrix elements can be explicitly written as

$$|V_{pq}| = |O_{1p}^l O_{1q}^\nu e^{i\alpha} + O_{2p}^l O_{2q}^\nu e^{i\beta} + O_{3p}^l O_{3q}^\nu|, \tag{8}$$

where p and q run over 1, 2, 3. We defined $\beta \equiv \phi_\nu - \phi_l$ and $\alpha \equiv (\varphi_\nu - \varphi_l) - \beta$. The mixing matrix V is entirely determined by the charged-lepton mass ratios (x_l, z_l) , the neutrino mass ratios (x_ν, z_ν) and two phases (α, β) . The current experimental data on neutrino oscillations will place restrictive constraints on these four unknown parameters (x_ν, z_ν) and (α, β) .

In the standard parametrization the lepton mixing matrix V is

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

Here $s_{ij} \equiv \sin \theta_{ij}$ and $c_{ij} \equiv \cos \theta_{ij}$, δ is the Dirac-type CP-violating phase and (ρ, σ) are the Majorana-type CP-violating phases. For the normal neutrino mass hierarchy the latest global-fit analysis yields at the 3σ level [6]:

$$\begin{aligned}
0.259 &\leq \sin^2 \theta_{12} \leq 0.359, \\
0.331 &\leq \sin^2 \theta_{23} \leq 0.637, \\
0.017 &\leq \sin^2 \theta_{13} \leq 0.031
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
6.99 \times 10^{-5} \text{ eV}^2 &\leq \delta m^2 \leq 8.18 \times 10^{-5} \text{ eV}^2, \\
2.19 \times 10^{-3} \text{ eV}^2 &\leq \Delta m^2 \leq 2.62 \times 10^{-3} \text{ eV}^2.
\end{aligned} \tag{11}$$

Table 1: The latest global-fit results of three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and two neutrino mass-squared differences $\delta m^2 \equiv m_2^2 - m_1^2$ and $\Delta m^2 \equiv m_3^2 - (m_1^2 + m_2^2)/2$ in the case of normal neutrino mass hierarchy [6].

Parameter	δm^2 (10^{-5} eV 2)	Δm^2 (10^{-3} eV 2)	θ_{12}	θ_{23}	θ_{13}
Best fit	7.54	2.43	33.6°	38.4°	8.9°
1 σ range	[7.32, 7.80]	[2.33, 2.49]	[32.6°, 34.8°]	[37.2°, 40.0°]	[8.5°, 9.4°]
2 σ range	[7.15, 8.00]	[2.27, 2.55]	[31.6°, 35.8°]	[36.2°, 42.0°]	[8.0°, 9.8°]
3 σ range	[6.99, 8.18]	[2.19, 2.62]	[30.6°, 36.8°]	[35.1°, 53.0°]	[7.5°, 10.2°]

Here the neutrino mass-squared differences are defined as $\delta m^2 \equiv m_2^2 - m_1^2$ and $\Delta m^2 \equiv m_3^2 - (m_1^2 + m_2^2)/2$. The best-fit values, 1 σ - and 2 σ -ranges of neutrino mixing parameters can be found in Table 1. Furthermore we have

$$R_\nu \equiv \frac{\delta m^2}{\Delta m^2} = \frac{z_\nu^2}{x_\nu^2} \cdot \frac{1 - x_\nu^2}{1 - z_\nu^2(1 + x_\nu^{-2})/2}, \quad (12)$$

and

$$\sin^2 \theta_{12} = \frac{|V_{e2}|^2}{1 - |V_{e3}|^2}, \quad \sin^2 \theta_{23} = \frac{|V_{\mu 2}|^2}{1 - |V_{e3}|^2}, \quad \sin^2 \theta_{13} = |V_{e3}|^2, \quad (13)$$

where $V_{\alpha i}$ should be identified as V_{ji} with $\alpha = e, \mu, \tau$ corresponding to $j = 1, 2, 3$, respectively.

From the global-fit data on neutrino mass-squared differences in Eq. (11) and the definition of R_ν in Eq. (12) one finds $0.027 < R_\nu < 0.037$ at the 3 σ level. If $z_\nu^2 \ll x_\nu^2 \ll 1$, then $R_\nu \approx z_\nu^2/x_\nu^2$ holds as a good approximation.

Due to the strong mass hierarchy of charged leptons the orthogonal matrix O_l is approximately

$$O_l \approx \begin{pmatrix} 1 & -x_l^{1/2} & +z_l^{3/2}x_l^{-1} \\ +x_l^{1/2} & 1 & +z_l^{1/2}x_l^{-1/2} \\ -z_l^{1/2} & -z_l^{1/2}x_l^{-1/2} & 1 \end{pmatrix}. \quad (14)$$

We obtain, using Eqs. (8) and (14):

$$|V_{e3}| < \sqrt{\frac{m_1}{m_2}} \cdot \left(\frac{\delta m^2}{\Delta m^2} \right)^{3/4} + \sqrt{\frac{m_e}{m_\mu}} \cdot \left(\frac{\delta m^2}{\Delta m^2} \right)^{1/4} + \sqrt{\frac{m_e}{m_\tau}}, \quad (15)$$

where we have assumed $z_\nu^2 \ll x_\nu^2 \ll 1$, i.e. $R_\nu \approx z_\nu^2/x_\nu^2$. For $x_\nu \approx 0.3$ and $R_\nu < 0.037$ we then arrive at $|V_{e3}| < 0.09$, which is in contradiction with the lower bound $|V_{e3}| > 0.13$ at the 3 σ level. It is clear from Eq. (15) that θ_{13} is highly suppressed, because the ratio of two neutrino mass-squared differences turns out to be small and the contribution from the charged-lepton sector is negligible.

Thus the scenario with both M_l and M_ν of the form in Eq. (3) has been ruled out by current experimental data on neutrino oscillations at the 3 σ level.

3 Canonical Seesaw Model

In order to accommodate the tiny neutrino masses, one can extend the standard model by introducing three right-handed singlet neutrinos. The Lagrangian relevant for lepton masses is

$$-\mathcal{L}_{\text{SS}} = \overline{l}_L Y_l H E_R + \overline{l}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \overline{N}_R^c M_R N_R + \text{h.c.}, \quad (16)$$

where l_L and $\tilde{H} \equiv i\sigma_2 H^*$ stand respectively for the lepton and Higgs doublets, E_R and N_R denote the charged-lepton and neutrino singlets, Y_l and Y_ν are the charged-lepton and Dirac neutrino Yukawa coupling matrices, and M_R is the mass matrix of right-handed neutrinos.

After the electroweak gauge symmetry breaking the charged-lepton and Dirac neutrino mass matrices are given by $M_l = Y_l v$ and $M_D = Y_\nu v$, with $v \approx 174$ GeV being the vacuum expectation value of the Higgs field. The effective mass matrix of the three light neutrinos is then determined by the seesaw formula $M_\nu = M_D M_R^{-1} M_D^T$ [7]. Given $\mathcal{O}(M_R) \sim 10^{14}$ GeV close to the grand-unified-theory scale and $\mathcal{O}(M_D) \sim 10^2$ GeV at the electroweak scale, the neutrino masses are in the sub-eV region. The smallness of neutrino masses can be ascribed to the heaviness of right-handed Majorana neutrinos. Although the seesaw mechanism can explain well the tiny neutrino masses, it cannot fix the structure of lepton mass matrices that determines the lepton masses and the flavor mixing pattern.

We assume that all the lepton mass matrices M_l , M_D and M_R have the texture zeros, given in Eq. (3), and denote the corresponding matrix elements by different subscripts $f = l, D, R$. Thus the effective neutrino mass matrix is

$$M_\nu = M_D M_R^{-1} M_D^T = \begin{pmatrix} 0 & C_\nu & 0 \\ C_\nu & \mathcal{D}_\nu & \mathcal{B}_\nu \\ 0 & \mathcal{B}_\nu & \mathcal{A}_\nu \end{pmatrix}, \quad (17)$$

where the matrix elements are given by

$$\begin{aligned} \mathcal{A}_\nu &= \frac{\mathcal{A}_D^2}{\mathcal{A}_R}, \\ \mathcal{C}_\nu &= \frac{\mathcal{C}_D^2}{\mathcal{C}_R}, \\ \mathcal{D}_\nu &= \frac{\mathcal{C}_D^2}{\mathcal{A}_R} \left(\frac{\mathcal{B}_D}{\mathcal{C}_D} - \frac{\mathcal{B}_R}{\mathcal{C}_R} \right)^2, \\ \mathcal{B}_\nu &= \frac{\mathcal{B}_D \mathcal{C}_D}{\mathcal{C}_R} + \frac{\mathcal{C}_D \mathcal{A}_D}{\mathcal{A}_R} \left(\frac{\mathcal{B}_D}{\mathcal{C}_D} - \frac{\mathcal{B}_R}{\mathcal{C}_R} \right). \end{aligned} \quad (18)$$

If the condition $\mathcal{B}_D/\mathcal{C}_D = \mathcal{B}_R/\mathcal{C}_R$ is satisfied, then $\mathcal{D}_\nu = 0$ [see Eq. (3)]. The matrix elements of M_ν individually follow the seesaw relation $\mathcal{A}_\nu = \mathcal{A}_D^2/\mathcal{A}_R$, $\mathcal{B}_\nu = \mathcal{B}_D^2/\mathcal{B}_R$, and $\mathcal{C}_\nu = \mathcal{C}_D^2/\mathcal{C}_R$ [8]. Although this seesaw-invariant scenario is phenomenologically interesting, it is disfavored by the neutrino oscillation data, as we have shown in the previous section. Another different model with texture zeros of the lepton mass matrices in the seesaw framework can be found in Ref. [9].

In order to obtain the leptonic mixing matrix, one must diagonalize both the charged-lepton and effective neutrino mass matrices. The charged-lepton mass matrix M_l can be diagonalized in the same way as before, while the neutrino mass matrix M_ν in Eq. (17) can be diagonalized as follows. For simplicity we assume $\arg[\mathcal{D}_\nu] = 2 \arg[\mathcal{B}_\nu]$, then M_ν can be decomposed as $M_\nu = P_\nu \overline{M}'_\nu P_\nu^T$ with

$$\overline{M}'_\nu = \begin{pmatrix} 0 & C_\nu & 0 \\ C_\nu & D_\nu & B_\nu \\ 0 & B_\nu & A_\nu \end{pmatrix}, \quad (19)$$

and $P_\nu = \text{Diag}\{e^{i(\varphi_\nu - \phi_\nu)}, e^{i\phi_\nu}, 1\}$, where $A_\nu = \mathcal{A}_\nu$, $B_\nu = |\mathcal{B}_\nu|$, $C_\nu = |\mathcal{C}_\nu|$, $D_\nu = |\mathcal{D}_\nu|$, $\varphi_\nu = \arg[\mathcal{C}_\nu]$ and $\phi_\nu = \arg[\mathcal{B}_\nu]$. The real and symmetric matrix \overline{M}'_ν can be diagonalized by an orthogonal transformation

$$O_\nu^T \overline{M}'_\nu O_\nu = \begin{pmatrix} \lambda_1^\nu & 0 & 0 \\ 0 & \lambda_2^\nu & 0 \\ 0 & 0 & \lambda_3^\nu \end{pmatrix}, \quad (20)$$

where $\lambda_1^\nu \lambda_2^\nu < 0$ follows from $\text{Det}[\overline{M}'_\nu] < 0$.

We shall discuss two different cases: $(\lambda_1^\nu, \lambda_2^\nu) = (+m_1, -m_2)$ and $(\lambda_1^\nu, \lambda_2^\nu) = (-m_1, +m_2)$. We take $(\lambda_1^\nu, \lambda_2^\nu) = (+m_1, -m_2)$ and $\lambda_3^\nu = m_3 > 0$. The other case can be discussed in a similar way. The four independent real parameters in \overline{M}'_ν cannot be expressed in terms of the three neutrino mass eigenvalues m_1, m_2 , and m_3 . Hence we define $r_\nu \equiv D_\nu/A_\nu$ and obtain

$$\begin{aligned} A_\nu &= (m_1 - m_2 + m_3)/(1 + r_\nu), \\ B_\nu &= \left[\frac{(r_\nu m_1 + m_2 - m_3)(m_1 + r_\nu m_2 + m_3)(m_1 - m_2 - r_\nu m_3)}{(m_1 - m_2 + m_3)(1 + r_\nu)^2} \right]^{1/2}, \\ C_\nu &= \left[\frac{m_1 m_2 m_3 (1 + r_\nu)}{m_1 - m_2 + m_3} \right]^{1/2}. \end{aligned} \quad (21)$$

The matrix elements of O_ν are

$$\begin{aligned} O_{11}^\nu &= + \left[\frac{x_\nu - z_\nu - r_\nu x_\nu^2}{(1 + x_\nu)(1 - z_\nu)(x_\nu - z_\nu + x_\nu z_\nu)} \right]^{1/2}, \\ O_{12}^\nu &= - \left[\frac{x_\nu^2(x_\nu + x_\nu z_\nu + r_\nu z_\nu)}{(1 + x_\nu)(x_\nu + z_\nu)(x_\nu - z_\nu + x_\nu z_\nu)} \right]^{1/2}, \\ O_{13}^\nu &= + \left[\frac{z_\nu^2(z_\nu - x_\nu z_\nu + r_\nu x_\nu)}{(1 - z_\nu)(x_\nu + z_\nu)(x_\nu - z_\nu + x_\nu z_\nu)} \right]^{1/2}, \\ O_{21}^\nu &= + \left[\frac{x_\nu - z_\nu + r_\nu x_\nu z_\nu}{(1 + x_\nu)(1 - z_\nu)(1 + r_\nu)} \right]^{1/2}, \\ O_{22}^\nu &= + \left[\frac{x_\nu + x_\nu z_\nu + r_\nu z_\nu}{(1 + x_\nu)(x_\nu + z_\nu)(1 + r_\nu)} \right]^{1/2}, \\ O_{23}^\nu &= + \left[\frac{z_\nu - x_\nu z_\nu + r_\nu x_\nu}{(1 - z_\nu)(x_\nu + z_\nu)(1 + r_\nu)} \right]^{1/2}, \\ O_{31}^\nu &= - \left[\frac{(x_\nu + x_\nu z_\nu + r_\nu z_\nu)(z_\nu - x_\nu z_\nu + r_\nu x_\nu)}{(1 + x_\nu)(1 - z_\nu)(x_\nu - z_\nu + x_\nu z_\nu)(1 + r_\nu)} \right]^{1/2}, \\ O_{32}^\nu &= - \left[\frac{(x_\nu - z_\nu - r_\nu x_\nu z_\nu)(z_\nu - x_\nu z_\nu + r_\nu x_\nu)}{(1 + x_\nu)(x_\nu + z_\nu)(x_\nu - z_\nu + x_\nu z_\nu)(1 + r_\nu)} \right]^{1/2}, \\ O_{33}^\nu &= + \left[\frac{(x_\nu - z_\nu - r_\nu x_\nu z_\nu)(x_\nu + x_\nu z_\nu + r_\nu z_\nu)}{(1 - z_\nu)(x_\nu + z_\nu)(x_\nu - z_\nu + x_\nu z_\nu)(1 + r_\nu)} \right]^{1/2}. \end{aligned} \quad (22)$$

The lepton mixing matrix V is given by Eq. (8), however, the matrix elements of O_ν should be replaced by those in Eq. (22). The mixing matrix V is entirely determined by the charged-lepton mass ratios (x_l, z_l) , the neutrino mass ratios (x_ν, z_ν) , the new parameter r_ν and two phases (α, β) . With the additional parameter r_ν we expect that the neutrino oscillation data can be explained. In the following we consider two simplified textures of M_ν and illustrate, how the texture zeros of the lepton mass matrices survive current experimental data:

- $D_\nu = m_2$, where m_2 denotes the mass of ν_2 . This assumption has conventionally been made in the study of the four-zero textures of quark [10] and lepton mass matrices [11].

- $D_\nu = A_\nu$. It is worthwhile to note that the 2-3 block of neutrino mass matrix M_ν in this case can be diagonalized by a maximal rotation angle. However, the mixing angle θ_{23} should not be maximal because of the moderate neutrino mass hierarchy and the correction from the charged-lepton sector.

As mentioned before, $D_\nu = 0$ if $\mathcal{B}_D/\mathcal{C}_D = \mathcal{B}_R/\mathcal{C}_R$ holds, so both the charged-lepton and effective neutrino mass matrices in this case are of the form, given in Eq. (3). This scenario is not consistent with the neutrino oscillation data, which indicates that the structure of Dirac neutrino mass matrix M_D should be quite different from that of the heavy Majorana neutrino mass matrix M_R . This seems to be more natural because the former is generally an arbitrary matrix, while the latter should be symmetric.

3.1 Case (A): $D_\nu = m_2$

In this case we obtain $r_\nu = z_\nu/(x_\nu - 2z_\nu + x_\nu z_\nu)$. Substituting r_ν into Eq. (21), one can calculate the other non-vanishing matrix elements of \overline{M}'_ν in Eq. (19)

$$\begin{aligned} A_\nu &= m_3 - 2m_2 + m_1, \\ B_\nu &= \left[\frac{(m_3 - 2m_2)(m_3 - m_2 + m_1)(2m_2 - m_1)}{m_3 - 2m_2 + m_1} \right]^{1/2}, \\ C_\nu &= \left[\frac{m_1 m_2 m_3}{m_3 - 2m_2 + m_1} \right]^{1/2}. \end{aligned} \quad (23)$$

The matrix elements of \overline{M}_l in Eq. (4) are given by

$$\begin{aligned} A_l &= m_\tau - m_\mu + m_e, \\ B_l &= \left[\frac{(m_\mu - m_e)(m_\tau - m_\mu)(m_\tau + m_e)}{m_\tau - m_\mu + m_e} \right]^{1/2}, \\ C_l &= \left[\frac{m_e m_\mu m_\tau}{m_\tau - m_\mu + m_e} \right]^{1/2}. \end{aligned} \quad (24)$$

Approximately one finds $A_l \approx m_\tau$, $B_l \approx \sqrt{m_\mu m_\tau}$, and $C_l \approx \sqrt{m_e m_\mu}$. Inserting r_ν into Eq. (22), we can obtain the orthogonal matrix O_ν and the lepton mixing matrix V , which consists of four parameters (x_ν, z_ν) and (α, β) . By using the current neutrino oscillation data, we find that **Case (A)** is successful at the 3σ level. The allowed regions of the neutrino mass ratios (x_ν, z_ν) and the phases (α, β) , together with those of the neutrino mixing parameters and other observables, are shown in Fig. 1. We note:

1. The allowed regions of (x_ν, z_ν) and (α, β) are given in the two plots in the first row of Fig. 1. We observe that $x_\nu \sim 0.5$ and $z_\nu \sim 0.1$ are typical values, so the neutrinos have a normal mass hierarchy $m_1 < m_2 < m_3$. Note that both the neutrino mass ratios (x_ν, z_ν) and the phase parameters (α, β) are restrictively constrained. Only a fraction of the parameter space around $\alpha = \pi$ and $\beta = \pi$ is allowed.
2. In the second row of Fig. 1 we show the predictions for the neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and the ratio of neutrino mass-squared differences R_ν . Although a relatively large θ_{13} can now

be achieved, an upper bound $\theta_{13} < 8.8^\circ$ exists, an even larger θ_{13} requires a large value of R_ν that is experimentally disfavored. Moreover $\theta_{12} > 32.6^\circ$ and $\theta_{23} < 46.5^\circ$ are predicted. In the same plots the best-fit values of $(\theta_{12}, \theta_{23})$ and (R_ν, θ_{13}) are represented by red solid squares, which are lying far away from the allowed parameter space. The 3σ , 2σ and 1σ ranges are bounded by the solid, dashed and dotted lines respectively. Therefore more precise measurements of neutrino mixing parameters are needed to verify or rule out the model under consideration.

3. The predictions for the Jarlskog invariant J and the Dirac-type and Majorana-type CP-violating phases (δ, ρ, σ) are given in the third row of Fig. 1. If the CP-violating phase takes the maximal value of $\delta = 0.18 \pi$, then the Jarlskog invariant J can reach 1.6%, which could hopefully be measured in future long-baseline neutrino oscillation experiments. On the other hand, two Majorana-type CP-violating phases are found to be $\rho \sim 0$ and $\sigma \sim \pi/2$. The relation $\sigma - \rho = \pi/2$ holds due to the imaginary unit arising from $\text{Det}[M_\nu] < 0$, and $\alpha \sim \beta \sim \pi$, as shown in Fig. 1.
4. The last row of Fig. 1 shows the predictions for the absolute neutrino mass m_3 , the effective neutrino mass $\langle m \rangle_\beta$ in the tritium beta decay, and the effective neutrino mass $\langle m \rangle_{\beta\beta}$ in the neutrinoless double-beta decays. One can observe that $\langle m \rangle_\beta < 1.0 \times 10^{-2}$ eV and $\langle m \rangle_{\beta\beta} < 2.4 \times 10^{-3}$ eV, both of which are difficult to be measured in the ongoing experiments. This is the usual situation for the case of normal neutrino mass hierarchy, in which the contribution from the heaviest neutrino mass eigenstate ν_3 is suppressed by the smallest mixing angle θ_{13} .

We conclude that **Case (A)** is compatible with the current neutrino oscillation data at the 3σ level. The future precision measurements of the neutrino mixing parameters can rule out this case if the experimental results finally converge to the current best-fit values.

Taking the typical values of $m_3 = 5.0 \times 10^{-2}$ eV and $(x_\nu, z_\nu) = (0.5, 0.1)$, one can calculate the other neutrino masses: $m_2 = 1.0 \times 10^{-2}$ eV and $m_1 = 5.0 \times 10^{-3}$ eV. One can also compute the matrix elements of \overline{M}'_ν and \overline{M}_l :

$$\begin{aligned} \overline{M}'_\nu &\approx 3.5 \times 10^{-2} \text{ eV} \cdot \begin{pmatrix} 0 & 0.24 & 0 \\ 0.24 & 0.29 & 0.69 \\ 0 & 0.69 & 1 \end{pmatrix}, \\ \overline{M}_l &\approx 1.67 \text{ GeV} \cdot \begin{pmatrix} 0 & 0.0045 & 0 \\ 0.0045 & 0 & 0.26 \\ 0 & 0.26 & 1 \end{pmatrix}. \end{aligned} \quad (25)$$

In order to see the structure of the heavy Majorana neutrino mass matrix M_R , we have to specify the structure of Dirac neutrino mass matrix M_D . Quite different from the four-zero textures of lepton mass matrices in the seesaw model, the Dirac neutrino mass matrix M_D is not arbitrary, but subject to the constraint relation

$$\frac{B_D}{C_D} = \frac{B_\nu}{C_\nu} + \frac{A_\nu}{C_\nu} \sqrt{\frac{D_\nu}{A_\nu}} \quad (26)$$

from Eq. (18), where we neglect the phases of the matrix elements for an order of magnitude estimate. Since M_D is of the form in Eq. (3), we write it in terms of its eigenvalues:

$$M_D = \begin{pmatrix} 0 & \sqrt{d_1 d_2} & 0 \\ \sqrt{d_1 d_2} & 0 & \sqrt{d_2 d_3} \\ 0 & \sqrt{d_2 d_3} & d_3 \end{pmatrix} \quad (27)$$

where $d_3 = m_t = 174$ GeV is the running top-quark mass at the electroweak scale. For illustration $d_1 : d_2 : d_3 = 1 : 5 : 25$ will be assumed to satisfy the constraint in Eq. (26). Thus M_R is found via the seesaw formula $M_R = M_D^T M_\nu^{-1} M_D$ to be

$$M_R \approx 8.7 \times 10^{14} \text{ GeV} \cdot \begin{pmatrix} 0 & 0.033 & 0 \\ 0.033 & 0 & 0.35 \\ 0 & 0.35 & 1 \end{pmatrix}. \quad (28)$$

It is obvious that both M_D and M_R cannot be strongly hierarchical, although the hierarchy of M_R is slightly stronger than that of M_D . With the help of Eq. (18) we find that the hierarchy of M_R is dictated by M_D and M_ν , since

$$\frac{B_R}{C_R} = \frac{B_D}{C_D} + \frac{A_D}{C_D} \sqrt{\frac{D_\nu}{A_\nu}}. \quad (29)$$

Note that the relation $B_\nu/C_\nu = B_D/C_D = B_R/C_R$ is reproduced if we set $D_\nu = 0$ in Eqs. (26) and (29). If the complex phases in M_R and M_D are included, the lepton number asymmetry can be produced in the CP-violating and out-of-equilibrium decays of heavy Majorana neutrinos in the early Universe [12]. Through the sphaleron processes, the lepton number asymmetry will be converted into baryon number asymmetry, explaining the matter-antimatter asymmetry in our Universe. We expect this elegant leptogenesis mechanism works well in the present case.

3.2 Case (B): $D_\nu = A_\nu$

Now we consider another example with $D_\nu = A_\nu$, namely $r_\nu = 1$. In this case the other non-vanishing matrix elements of \overline{M}'_ν in Eq. (19) are:

$$\begin{aligned} A_\nu &= \frac{1}{2}(m_1 - m_2 + m_3), \\ B_\nu &= \frac{1}{2} \left[\frac{(m_1 + m_2 - m_3)(m_1 + m_2 + m_3)(m_1 - m_2 - m_3)}{(m_1 - m_2 + m_3)} \right]^{1/2}, \\ C_\nu &= \left[\frac{2m_1 m_2 m_3}{m_1 - m_2 + m_3} \right]^{1/2}. \end{aligned} \quad (30)$$

The matrix elements of \overline{M}_l are still given by Eq. (24). Inserting $r_\nu = 1$ into Eq. (22), we obtain the orthogonal matrix O_ν and the lepton mixing matrix V , which is completely determined by four parameters (x_ν, z_ν) and (α, β) . With the help of current neutrino oscillation data, we have found **Case (B)** is perfectly consistent with experimental data – even the best-fit values of neutrino mixing parameters can be reproduced. Our numerical results are shown in Fig. 2. We note:

1. The allowed parameter space of the neutrino mass ratios (x_ν, z_ν) and the phase parameters (α, β) is given in the plots in the first row of Fig. 2. The typical values of the neutrino mass ratios are $x_\nu = 0.32$ and $z_\nu = 0.06$, indicating a moderate mass hierarchy $m_1 : m_2 : m_3 \approx 1 : 3 : 15$. Note that $\beta = \pi$ is experimentally disallowed, and only a fraction of parameter space around $\alpha \sim \beta \sim \pi/2$ (or $3\pi/2$) is favored.
2. The predictions for three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ and the ratio of the neutrino mass-squared differences R_ν are shown in the second row of Fig. 2. One can observe that even the best-fit values (red squares) and the 1σ ranges (regions between the dotted lines) of these

observables can be achieved. To understand this result, we calculate $|V_{e3}|$ in the same way as in Eq. (15):

$$|V_{e3}| < \sqrt{\frac{m_1}{m_2}} \left(\frac{\delta m^2}{\Delta m^2} \right)^{1/2} + \frac{1}{\sqrt{2}} \left(\sqrt{\frac{m_e}{m_\mu}} + \sqrt{\frac{m_e}{m_\tau}} \right), \quad (31)$$

where we have assumed $z_\nu^2 \ll x_\nu^2 \ll 1$, i.e. $R_\nu \approx z_\nu^2/x_\nu^2$. Given $x_\nu \approx 0.3$ and $R_\nu < 0.037$ at the 3σ level, we arrive at $|V_{e3}| < 0.167$, which is well compatible with the current neutrino oscillation data.

3. As shown in the plots in the last two rows of Fig. 2, the predictions for the leptonic CP violation, the effective neutrino masses in the tritium decay and the neutrinoless double-beta decay are similar to those in **Case (A)**. It is possible to discover leptonic CP violation in the future long-baseline neutrino oscillation experiments, while it is quite challenging to determine the Majorana neutrino mass in the neutrinoless double-beta decay.

Taking the typical values of $m_3 = 5.0 \times 10^{-2}$ eV and $(x_\nu, z_\nu) = (0.32, 0.06)$, one can determine the other neutrino masses: $m_2 = 9.4 \times 10^{-3}$ eV and $m_1 = 3.0 \times 10^{-3}$ eV. For an order of magnitude estimate we neglect the complex phases in the lepton mass matrices. The neutrino and charged-lepton mass matrices are:

$$\begin{aligned} \overline{M}'_\nu &\approx 2.2 \times 10^{-2} \text{ eV} \cdot \begin{pmatrix} 0 & 0.37 & 0 \\ 0.37 & 1 & 1.26 \\ 0 & 1.26 & 1 \end{pmatrix}, \\ \overline{M}_l &\approx 1.67 \text{ GeV} \cdot \begin{pmatrix} 0 & 0.0045 & 0 \\ 0.0045 & 0 & 0.26 \\ 0 & 0.26 & 1 \end{pmatrix}. \end{aligned} \quad (32)$$

Note that $B_\nu > A_\nu$ holds for the effective neutrino mass matrix. In order to see the structure of M_R , we assume M_D to be the same as in Eq. (27), but with $d_1 : d_2 : d_3 = 1 : 6 : 36$, and obtain

$$M_R \approx 1.4 \times 10^{15} \text{ GeV} \cdot \begin{pmatrix} 0 & 0.01 & 0 \\ 0.01 & 0 & 0.25 \\ 0 & 0.25 & 1 \end{pmatrix}. \quad (33)$$

Again both M_D and M_R cannot have a strong hierarchy in the matrix elements, due to the moderate neutrino mass hierarchy and the constraint conditions in Eqs. (26) and (29).

4 Summary

Taking into account the new results of the neutrino oscillation experiments, we study the validity of the texture zeros in the lepton mass matrices. Due to the large angle θ_{13} the original scenario, in which both the charged-lepton mass matrix M_l and the effective neutrino mass matrix M_ν have three texture zeros, is inconsistent with current neutrino oscillation data. In the canonical seesaw model we apply the texture zeros to the charged-lepton mass matrix M_l , the Dirac neutrino mass matrix M_D , and the heavy Majorana neutrino mass matrix M_R . We present two phenomenologically interesting models that are compatible with the experimental data. The nonmaximal θ_{23} and unsuppressed θ_{13} can naturally be accommodated. The neutrinos have a normal mass hierarchy, and the effective

neutrino masses $\langle m \rangle_\beta$ in the tritium beta decay and $\langle m \rangle_{\beta\beta}$ in the neutrinoless double-beta decays are found to be at the meV level. The future precision measurements of the neutrino mixing parameters and the possible discovery of leptonic CP violation will allow us to distinguish between these two models and to test their validity.

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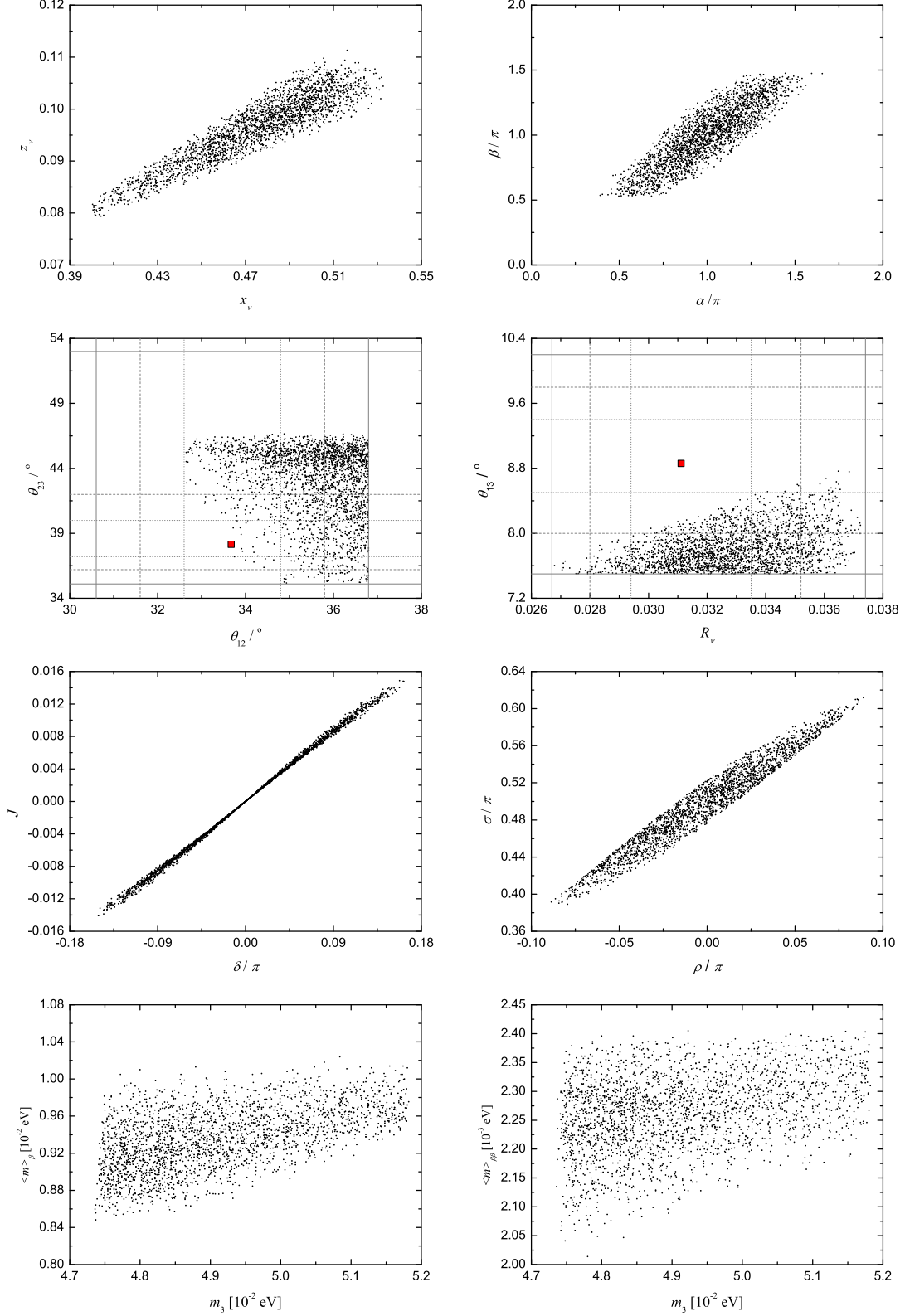


Figure 1: **Case (A)** with $D_\nu = m_2$: Allowed regions of the neutrino mass ratios (x_ν, z_ν) and two phase parameters (α, β), where the 3σ ranges of neutrino mixing angles and mass-squared differences are taken as input [6]. The predictions for three neutrino mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), the CP-violating phases (δ, ρ, σ), the Jarlskog invariant J , the effective neutrino masses in the tritium beta decay $\langle m \rangle_\beta$ and in the neutrinoless double-beta decays $\langle m \rangle_{\beta\beta}$ are also shown.

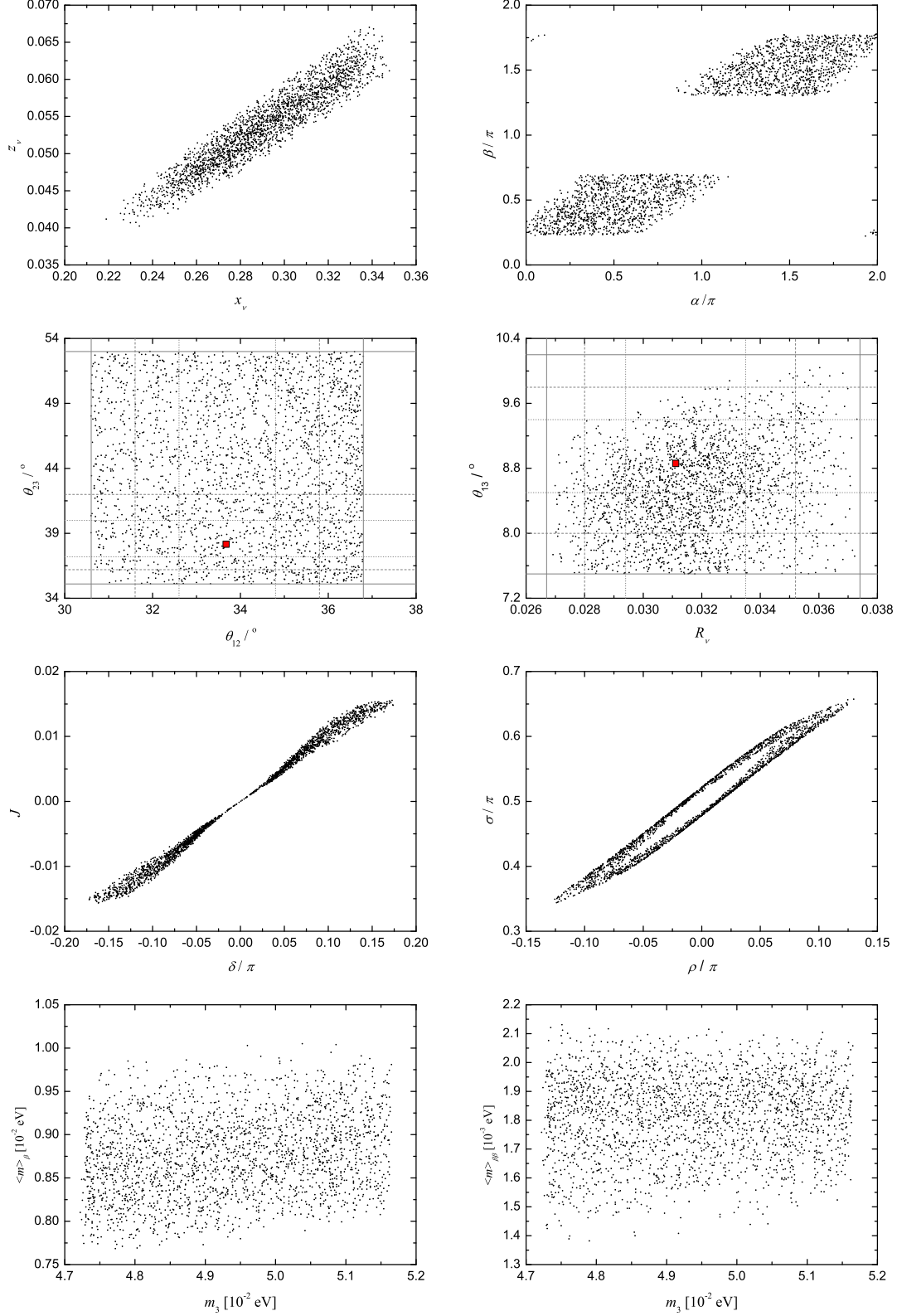


Figure 2: **Case (B)** with $D_\nu = A_\nu$: Allowed regions of the neutrino mass ratios (x_ν, z_ν) and two phase parameters (α, β) , where the 3σ ranges of neutrino mixing angles and mass-squared differences are taken as input [6]. The predictions for three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, the CP-violating phases (δ, ρ, σ) , the Jarlskog invariant J , the effective neutrino masses in the tritium beta decay $\langle m \rangle_\beta$ and in the neutrinoless double-beta decays $\langle m \rangle_{\beta\beta}$ are also shown.